**Lower Tail Test of Population Mean with Known Variance**

The null hypothesis of the **lower tail test of the population mean**can be expressed as follows:

![μ ≥ μ0
]()

where *μ*0 is a hypothesized lower bound of the true population mean *μ*.

Let us define the test statistic *z*in terms of the [sample mean](http://www.r-tutor.com/node/35), the sample size and the [population standard deviation](http://www.r-tutor.com/node/43) *σ*:

z = ¯x−√μ0-
    σ∕  n


Then the null hypothesis of the lower tail test is to be *rejected*if *z*≤−*zα* , where *zα* is the 100(1 − *α*) [percentile](http://www.r-tutor.com/node/38) of the [standard normal distribution](http://www.r-tutor.com/node/58).

**Problem**

Suppose the manufacturer claims that the mean lifetime of a light bulb is more than 10,000 hours. In a sample of 30 light bulbs, it was found that they only last 9,900 hours on average. Assume the population standard deviation is 120 hours. At .05 significance level, can we reject the claim by the manufacturer?

**Solution**

The null hypothesis is that *μ*≥ 10000. We begin with computing the test statistic.

> xbar = 9900            # sample mean   
> mu0 = 10000            # hypothesized value   
> sigma = 120            # population standard deviation   
> n = 30                 # sample size   
> z = (xbar−mu0)/(sigma/sqrt(n))   
> z                      # test statistic   
[1] −4.5644

We then compute the critical value at .05 significance level.

> alpha = .05   
> z.alpha = qnorm(1−alpha)   
> −z.alpha               # critical value   
[1] −1.6449

**Answer**

The test statistic -4.5644 is less than the critical value of -1.6449. Hence, at .05 significance level, we reject the claim that mean lifetime of a light bulb is above 10,000 hours.

**Alternative Solution**

Instead of using the critical value, we apply the pnorm function to compute the lower tail **p-value**of the test statistic. As it turns out to be less than the .05 significance level, we reject the null hypothesis that *μ*≥ 10000.

> pval = pnorm(z)   
> pval                   # lower tail p−value   
[1] 2.5052e−06

**Upper Tail Test of Population Mean with Known Variance**

The null hypothesis of the **upper tail test of the population mean**can be expressed as follows:

![μ ≤ μ0
]()

where *μ*0 is a hypothesized upper bound of the true population mean *μ*.

Let us define the test statistic *z*in terms of the [sample mean](http://www.r-tutor.com/node/35), the sample size and the [population standard deviation](http://www.r-tutor.com/node/43) *σ*:

    ¯x− μ0
z = σ∕√n--


Then the null hypothesis of the upper tail test is to be *rejected*if *z*≥ *zα* , where *zα* is the 100(1 − *α*) [percentile](http://www.r-tutor.com/node/38) of the [standard normal distribution](http://www.r-tutor.com/node/58).

**Problem**

Suppose the food label on a cookie bag states that there is at most 2 grams of saturated fat in a single cookie. In a sample of 35 cookies, it is found that the mean amount of saturated fat per cookie is 2.1 grams. Assume that the population standard deviation is 0.25 grams. At .05 significance level, can we reject the claim on food label?

**Solution**

The null hypothesis is that *μ*≤ 2. We begin with computing the test statistic.

> xbar = 2.1             # sample mean   
> mu0 = 2                # hypothesized value   
> sigma = 0.25           # population standard deviation   
> n = 35                 # sample size   
> z = (xbar−mu0)/(sigma/sqrt(n))   
> z                      # test statistic   
[1] 2.3664

We then compute the critical value at .05 significance level.

> alpha = .05   
> z.alpha = qnorm(1−alpha)   
> z.alpha                # critical value   
[1] 1.6449

**Answer**

The test statistic 2.3664 is greater than the critical value of 1.6449. Hence, at .05 significance level, we reject the claim that there is at most 2 grams of saturated fat in a cookie.

**Alternative Solution**

Instead of using the critical value, we apply the pnorm function to compute the upper tail **p-value**of the test statistic. As it turns out to be less than the .05 significance level, we reject the null hypothesis that *μ*≤ 2.

> pval = pnorm(z, lower.tail=FALSE)   
> pval                   # upper tail p−value   
[1] 0.0089802

**Two-Tailed Test of Population Mean with Known Variance**

The null hypothesis of the **two-tailed test of the population mean**can be expressed as follows:

![μ = μ0
]()

where *μ*0 is a hypothesized value of the true population mean *μ*.

Let us define the test statistic *z*in terms of the [sample mean](http://www.r-tutor.com/node/35), the sample size and the [population standard deviation](http://www.r-tutor.com/node/43) *σ*:

    ¯x− μ0
z = σ∕√n--


Then the null hypothesis of the two-tailed test is to be *rejected*if *z*≤−*zα∕*2 or *z*≥ *zα∕*2 , where *zα∕*2 is the 100(1 − *α∕*2) [percentile](http://www.r-tutor.com/node/38) of the [standard normal distribution](http://www.r-tutor.com/node/58).

**Problem**

Suppose the mean weight of King Penguins found in an Antarctic colony last year was 15.4 kg. In a sample of 35 penguins same time this year in the same colony, the mean penguin weight is 14.6 kg. Assume the population standard deviation is 2.5 kg. At .05 significance level, can we reject the null hypothesis that the mean penguin weight does not differ from last year?

**Solution**

The null hypothesis is that *μ*= 15*.*4. We begin with computing the test statistic.

> xbar = 14.6            # sample mean   
> mu0 = 15.4             # hypothesized value   
> sigma = 2.5            # population standard deviation   
> n = 35                 # sample size   
> z = (xbar−mu0)/(sigma/sqrt(n))   
> z                      # test statistic   
[1] −1.8931

We then compute the critical values at .05 significance level.

> alpha = .05   
> z.half.alpha = qnorm(1−alpha/2)   
> c(−z.half.alpha, z.half.alpha)   
[1] −1.9600  1.9600

**Answer**

The test statistic -1.8931 lies between the critical values -1.9600 and 1.9600. Hence, at .05 significance level, we do *not*reject the null hypothesis that the mean penguin weight does not differ from last year.

**Alternative Solution**

Instead of using the critical value, we apply the pnorm function to compute the two-tailed **p-value**of the test statistic. It doubles the *lower*tail p-value as the sample mean is *less*than the hypothesized value. Since it turns out to be greater than the .05 significance level, we do *not*reject the null hypothesis that *μ*= 15*.*4.

> pval = 2 ∗ pnorm(z)    # lower tail   
> pval                   # two−tailed p−value   
[1] 0.058339

**Lower Tail Test of Population Mean with Unknown Variance**

The null hypothesis of the **lower tail test of the population mean**can be expressed as follows:

![μ ≥ μ0
]()

where *μ*0 is a hypothesized lower bound of the true population mean *μ*.

Let us define the test statistic *t*in terms of the [sample mean](http://www.r-tutor.com/node/35), the sample size and the [sample standard deviation](http://www.r-tutor.com/node/43) *s*:

    ¯x− μ0
t = s∕√n--


Then the null hypothesis of the lower tail test is to be *rejected*if *t*≤−*tα* , where *tα* is the 100(1 − *α*) [percentile](http://www.r-tutor.com/node/38) of the [Student t distribution](http://www.r-tutor.com/node/59) with *n*− 1 degrees of freedom.

**Problem**

Suppose the manufacturer claims that the mean lifetime of a light bulb is more than 10,000 hours. In a sample of 30 light bulbs, it was found that they only last 9,900 hours on average. Assume the sample standard deviation is 125 hours. At .05 significance level, can we reject the claim by the manufacturer?

**Solution**

The null hypothesis is that *μ*≥ 10000. We begin with computing the test statistic.

> xbar = 9900            # sample mean   
> mu0 = 10000            # hypothesized value   
> s = 125                # sample standard deviation   
> n = 30                 # sample size   
> t = (xbar−mu0)/(s/sqrt(n))   
> t                      # test statistic   
[1] −4.3818

We then compute the critical value at .05 significance level.

> alpha = .05   
> t.alpha = qt(1−alpha, df=n−1)   
> −t.alpha               # critical value   
[1] −1.6991

**Answer**

The test statistic -4.3818 is less than the critical value of -1.6991. Hence, at .05 significance level, we can reject the claim that mean lifetime of a light bulb is above 10,000 hours.

**Alternative Solution**

Instead of using the critical value, we apply the pt function to compute the lower tail **p-value**of the test statistic. As it turns out to be less than the .05 significance level, we reject the null hypothesis that *μ*≥ 10000.

> pval = pt(t, df=n−1)   
> pval                   # lower tail p−value   
[1] 7.035e−05

**Upper Tail Test of Population Mean with Unknown Variance**

The null hypothesis of the **upper tail test of the population mean**can be expressed as follows:

![μ ≤ μ0
]()

where *μ*0 is a hypothesized upper bound of the true population mean *μ*.

Let us define the test statistic *t*in terms of the [sample mean](http://www.r-tutor.com/node/35), the sample size and the [sample standard deviation](http://www.r-tutor.com/node/43) *s*:

    ¯x− μ0
t = s∕√n--


Then the null hypothesis of the upper tail test is to be *rejected*if *t*≥ *tα* , where *tα* is the 100(1 − *α*) [percentile](http://www.r-tutor.com/node/38) of the [Student t distribution](http://www.r-tutor.com/node/59) with *n*− 1 degrees of freedom.

**Problem**

Suppose the food label on a cookie bag states that there is at most 2 grams of saturated fat in a single cookie. In a sample of 35 cookies, it is found that the mean amount of saturated fat per cookie is 2.1 grams. Assume that the sample standard deviation is 0.3 gram. At .05 significance level, can we reject the claim on food label?

**Solution**

The null hypothesis is that *μ*≤ 2. We begin with computing the test statistic.

> xbar = 2.1             # sample mean   
> mu0 = 2                # hypothesized value   
> s = 0.3                # sample standard deviation   
> n = 35                 # sample size   
> t = (xbar−mu0)/(s/sqrt(n))   
> t                      # test statistic   
[1] 1.9720

We then compute the critical value at .05 significance level.

> alpha = .05   
> t.alpha = qt(1−alpha, df=n−1)   
> t.alpha                # critical value   
[1] 1.6991

**Answer**

The test statistic 1.9720 is greater than the critical value of 1.6991. Hence, at .05 significance level, we can reject the claim that there is at most 2 grams of saturated fat in a cookie.

**Alternative Solution**

Instead of using the critical value, we apply the pt function to compute the upper tail **p-value**of the test statistic. As it turns out to be less than the .05 significance level, we reject the null hypothesis that *μ*≤ 2.

> pval = pt(t, df=n−1, lower.tail=FALSE)   
> pval                   # upper tail p−value   
[1] 0.028393

**Two-Tailed Test of Population Mean with Unknown Variance**

The null hypothesis of the **two-tailed test of the population mean**can be expressed as follows:

![μ = μ0
]()

where *μ*0 is a hypothesized value of the true population mean *μ*.

Let us define the test statistic *t*in terms of the [sample mean](http://www.r-tutor.com/node/35), the sample size and the [sample standard deviation](http://www.r-tutor.com/node/43) *s*:

    ¯x− μ0
t = s∕√n--


Then the null hypothesis of the two-tailed test is to be *rejected*if *t*≤−*tα∕*2 or *t*≥ *tα∕*2 , where *tα∕*2is the 100(1 − *α*) [percentile](http://www.r-tutor.com/node/38) of the [Student t distribution](http://www.r-tutor.com/node/59) with *n*− 1 degrees of freedom.

**Problem**

Suppose the mean weight of King Penguins found in an Antarctic colony last year was 15.4 kg. In a sample of 35 penguins same time this year in the same colony, the mean penguin weight is 14.6 kg. Assume the sample standard deviation is 2.5 kg. At .05 significance level, can we reject the null hypothesis that the mean penguin weight does not differ from last year?

**Solution**

The null hypothesis is that *μ*= 15*.*4. We begin with computing the test statistic.

> xbar = 14.6            # sample mean   
> mu0 = 15.4             # hypothesized value   
> s = 2.5                # sample standard deviation   
> n = 35                 # sample size   
> t = (xbar−mu0)/(s/sqrt(n))   
> t                      # test statistic   
[1] −1.8931

We then compute the critical values at .05 significance level.

> alpha = .05   
> t.half.alpha = qt(1−alpha/2, df=n−1)   
> c(−t.half.alpha, t.half.alpha)   
[1] −2.0322  2.0322

**Answer**

The test statistic -1.8931 lies between the critical values -2.0322, and 2.0322. Hence, at .05 significance level, we do *not*reject the null hypothesis that the mean penguin weight does not differ from last year.

**Alternative Solution**

Instead of using the critical value, we apply the pt function to compute the two-tailed **p-value**of the test statistic. It doubles the *lower*tail p-value as the sample mean is *less*than the hypothesized value. Since it turns out to be greater than the .05 significance level, we do *not*reject the null hypothesis that *μ*= 15*.*4.

> pval = 2 ∗ pt(t, df=n−1)  # lower tail   
> pval                      # two−tailed p−value   
[1] 0.066876

**Lower Tail Test of Population Proportion**

The null hypothesis of the **lower tail test about population proportion**can be expressed as follows:

![p ≥ p0
]()

where *p*0 is a hypothesized lower bound of the true population proportion *p*.

Let us define the test statistic *z*in terms of the sample proportion and the sample size:

        ¯p− p0
z = ∘------------
      p0(1− p0)∕n


Then the null hypothesis of the lower tail test is to be *rejected*if *z*≤−*zα* , where *zα* is the 100(1 − *α*) [percentile](http://www.r-tutor.com/node/38) of the [standard normal distribution](http://www.r-tutor.com/node/58).

**Problem**

Suppose 60% of citizens voted in last election. 85 out of 148 people in a telephone survey said that they voted in current election. At 0.5 significance level, can we reject the null hypothesis that the proportion of voters in the population is above 60% this year?

**Solution**

The null hypothesis is that *p*≥ 0*.*6. We begin with computing the test statistic.

> pbar = 85/148          # sample proportion   
> p0 = .6                # hypothesized value   
> n = 148                # sample size   
> z = (pbar−p0)/sqrt(p0∗(1−p0)/n)   
> z                      # test statistic   
[1] −0.6376

We then compute the critical value at .05 significance level.

> alpha = .05   
> z.alpha = qnorm(1−alpha)   
> −z.alpha               # critical value   
[1] −1.6449

**Answer**

The test statistic -0.6376 is *not*less than the critical value of -1.6449. Hence, at .05 significance level, we do *not*reject the null hypothesis that the proportion of voters in the population is above 60% this year.

**Alternative Solution 1**

Instead of using the critical value, we apply the pnorm function to compute the lower tail **p-value**of the test statistic. As it turns out to be greater than the .05 significance level, we do not reject the null hypothesis that *p*≥ 0*.*6.

> pval = pnorm(z)   
> pval                   # lower tail p−value   
[1] 0.26187

**Alternative Solution 2**

We apply the prop.test function to compute the p-value directly. The Yates continuity correction is disabled for pedagogical reasons.

> prop.test(85, 148, p=.6, alt="less", correct=FALSE)   
   
        1−sample proportions test without continuity   
        correction   
   
data:  85 out of 148, null probability 0.6   
X−squared = 0.4065, df = 1, p−value = 0.2619   
alternative hypothesis: true p is less than 0.6   
95 percent confidence interval:   
 0.0000 0.63925   
sample estimates:   
      p   
0.57432

**Upper Tail Test of Population Proportion**

The null hypothesis of the **upper tail test about population proportion**can be expressed as follows:

![p ≤ p0
]()

where *p*0 is a hypothesized upper bound of the true population proportion *p*.

Let us define the test statistic *z*in terms of the sample proportion and the sample size:

        ¯p− p0
z = ∘------------
      p0(1− p0)∕n


Then the null hypothesis of the upper tail test is to be *rejected*if *z*≥ *zα* , where *zα* is the 100(1 − *α*) [percentile](http://www.r-tutor.com/node/38) of the [standard normal distribution](http://www.r-tutor.com/node/58).

**Problem**

Suppose that 12% of apples harvested in an orchard last year was rotten. 30 out of 214 apples in a harvest sample this year turns out to be rotten. At .05 significance level, can we reject the null hypothesis that the proportion of rotten apples in harvest stays below 12% this year?

**Solution**

The null hypothesis is that *p*≤ 0*.*12. We begin with computing the test statistic.

> pbar = 30/214          # sample proportion   
> p0 = .12               # hypothesized value   
> n = 214                # sample size   
> z = (pbar−p0)/sqrt(p0∗(1−p0)/n)   
> z                      # test statistic   
[1] 0.90875

We then compute the critical value at .05 significance level.

> alpha = .05   
> z.alpha = qnorm(1−alpha)   
> z.alpha                # critical value   
[1] 1.6449

**Answer**

The test statistic 0.90875 is *not*greater than the critical value of 1.6449. Hence, at .05 significance level, we do *not*reject the null hypothesis that the proportion of rotten apples in harvest stays below 12% this year.

**Alternative Solution 1**

Instead of using the critical value, we apply the pnorm function to compute the upper tail **p-value**of the test statistic. As it turns out to be greater than the .05 significance level, we do not reject the null hypothesis that *p*≤ 0*.*12.

> pval = pnorm(z, lower.tail=FALSE)   
> pval                   # upper tail p−value   
[1] 0.18174

**Alternative Solution 2**

We apply the prop.test function to compute the p-value directly. The Yates continuity correction is disabled for pedagogical reasons.

> prop.test(30, 214, p=.12, alt="greater", correct=FALSE)   
   
        1−sample proportions test without continuity   
        correction   
   
data:  30 out of 214, null probability 0.12   
X−squared = 0.8258, df = 1, p−value = 0.1817   
alternative hypothesis: true p is greater than 0.12   
95 percent confidence interval:   
 0.10563 1.00000   
sample estimates:   
      p   
0.14019

**Two-Tailed Test of Population Proportion**

The null hypothesis of the **two-tailed test about population proportion**can be expressed as follows:

![p = p0
]()

where *p*0 is a hypothesized value of the true population proportion *p*.

Let us define the test statistic *z*in terms of the sample proportion and the sample size:

        ¯p− p0
z = ∘------------
      p0(1− p0)∕n


Then the null hypothesis of the two-tailed test is to be *rejected*if *z*≤−*zα∕*2 or *z*≥ *zα∕*2 , where *zα∕*2 is the 100(1 − *α*) [percentile](http://www.r-tutor.com/node/38) of the [standard normal distribution](http://www.r-tutor.com/node/58).

**Problem**

Suppose a coin toss turns up 12 heads out of 20 trials. At .05 significance level, can one reject the null hypothesis that the coin toss is fair?

**Solution**

The null hypothesis is that *p*= 0*.*5. We begin with computing the test statistic.

> pbar = 12/20           # sample proportion   
> p0 = .5                # hypothesized value   
> n = 20                 # sample size   
> z = (pbar−p0)/sqrt(p0∗(1−p0)/n)   
> z                      # test statistic   
[1] 0.89443

We then compute the critical values at .05 significance level.

> alpha = .05   
> z.half.alpha = qnorm(1−alpha/2)   
> c(−z.half.alpha, z.half.alpha)   
[1] −1.9600  1.9600

**Answer**

The test statistic 0.89443 lies between the critical values -1.9600 and 1.9600. Hence, at .05 significance level, we do *not*reject the null hypothesis that the coin toss is fair.

**Alternative Solution 1**

Instead of using the critical value, we apply the pnorm function to compute the two-tailed **p-value**of the test statistic. It doubles the *upper*tail p-value as the sample proportion is *greater*than the hypothesized value. Since it turns out to be greater than the .05 significance level, we do not reject the null hypothesis that *p*= 0*.*5.

> pval = 2 ∗ pnorm(z, lower.tail=FALSE)  # upper tail   
> pval                   # two−tailed p−value   
[1] 0.37109

**Alternative Solution 2**

We apply the prop.test function to compute the p-value directly. The Yates continuity correction is disabled for pedagogical reasons.

> prop.test(12, 20, p=0.5, correct=FALSE)   
   
        1−sample proportions test without continuity   
        correction   
   
data:  12 out of 20, null probability 0.5   
X−squared = 0.8, df = 1, p−value = 0.3711   
alternative hypothesis: true p is not equal to 0.5   
95 percent confidence interval:   
  0.38658 0.78119   
sample estimates:   
  p   
0.6